Journal of Applied Econometrics and Statistics
Vol. 3, No. 1-2, 2024, pp. 79-95
https://DOI:10.47509/JAES.2024.v03i01-2.05

ARF INDIA
Academic Open Access Publishing
www.arfjournals.com

Asymmetry Volatility Modeling of Nigerian Stock Prices with Some Selected Distribution of Innovations

[1]Adenomo M.O., [2*]Awogbemi C.A., [3]Ilori A.K., [4]Shitu D.A., [5]Dayo V.K., [6]Chajire B.P., [7]Sani Z.S., [8]Nwikpe B.J., [9]Tanimu M. and [10]Paul V. B.

[1]*Department of Statistics, Nasarawa State University, Keffi, Nigeria*
[2,3]*Statistics Programme, National Mathematical Centre, Abuja, Nigeria*
[4]*Department of Statistics, Abubakar Tafawa Balewa University, Bauchi, Nigeria*
[5]*Department of Statistics, University of Abuja, Abuja, Nigeria*
[6]*Department of Mathematical Sciences, Gombe State University, Gombe, Nigeria*
[7]*Department of Statistics, Kano University of Science and Technology, Kano, Nigeria*
[8]*Department of Statistics, Ignatius Ajuru University of Education, Port Harcourt, Nigeria*
[9]*Department of Statistics, Federal University of Technology, Minna, Nigeria*
[10]*Department of Mathematics, Rivers State University, Port- Harcourt, Nigeria*
[*]*Corresponding Author E-mail: awogbemiadeyeye@yahoo.com*

Received: 28 May 2024; Revised: 15 June 2024;
Accepted 23 June 2024; Publication: 30 December 2024

Abstract: This study investigated the performance of EGARCH model for three different innovations (student's t, normal, and skewed innovations) using Nigerian daily stock price series from 30/01/2012 to 03/10/2024, yielding a total of 3139 observations. The aim was to determine the innovation distribution that best captures the asymmetry and kurtosis exhibited by the returns on financial data. The descriptive properties of the series revealed that the distribution of returns for the stock prices was skewed and leptokurtic. The unit root test was carried out using the augmented Dickey-Fuller (ADF) test, and the result revealed that the returns on the series was stationarity. The ARCH LM-test detected the presence of ARCH effects, which justified the use of the GARCH model. The mean equation was estimated, and EGARCH-X (1,1) model was fitted to the data, using different innovations. The findings of the study revealed that EGARCH (1,1) model with skewed Student's t-distribution gave the overall best fit, with the lowest AIC (-8.445) and the highest log-likelihood (13252.37). Findings of the study further revealed that the forecast from EGARCH (1,1) model with the skewed student's t-distribution has strong numerical accuracy with low RMSE (0.004413) and MAE (0.002907), indicating that the model's predictions are close to the actual values. From the findings of this study, it was deduced that the selection of suitable innovations in financial volatility modelling is pertinent for an appropriate forecast of the financial market.

Keywords: Asymmetry, E-GARCH, Stock Price, Volatility, Innovation Distributions.

---

1. INTRODUCTION

Modelling and forecasting the volatility of financial time series (i.e. asset returns) has become a fertile area of research lately in finance. The concept has received a considerable attention from economic practitioners, including academics. Volatility modelling has been an important integral of many economic and financial policy implementations, like portfolio optimization, risk management and asset pricing (Marobhe, 2020). The volatility of stock return is a measure of dispersion around the average return of a security or an index. Modeling volatility forms a vital part of designing investment plans to reduce risk and improve stock returns. It is also very useful in securities and options pricing. However, its importance is not only confined to investors and other market participants, but also to the overall economy (Zakaria, Abdalla, & Winker, 2017).

Investigating behaviors of stock returns is crucial to stock market participants as variation of returns from expectation could mean huge losses or gain and so, one of the most fascinating tasks is creating reliable and efficient models for accurate stock market price prediction (Emenyonu, Osu, & Olunkwa, 2023). It is now feasible to extract, store, process, and analyze large volumes of diversely-contented stock market data effectively and efficiently, thanks to the exponential rate of advancement and evolution of complex models and the availability of fast computing platforms. Distinct economic sectors display unique patterns of fluctuations in their stock prices. These sectors also differ from one another with respect to their trend patterns, seasonal characteristics, and time series randomness (Fateye, Ajayi, & Ajayi, 2022). While the primary tenet of the efficient market hypothesis has been the unpredictability of the stock market, research aiming to support or refute the idea has examined a variety of essential traits of various equities and produced a range of conclusions. Examining patterns from past data empowers investors on how to expect future price changes and manage investment risks effectively (Mohammed, Umar, & Adams, 2022). Stock price volatility is a critical concept in finance, representing the degree of variation in the price of a stock over time. Understanding and modeling this volatility is essential for risk management, portfolio optimization, and derivative pricing (Yeasin, Narain Singh, Lama, & Kumar Paul, 2020). Time series models are particularly useful for this purpose, as they analyze historical price data to identify patterns and forecast future price movements (Khan & Abdullah, 2019). This study utilized the asymmetric GARCH to model the volatility of Nigerian financial stock market using some selected innovation distributions (normal, Student's t, and skewed Student's t-distributions). The study analysed the trend, compared the various performance of the model with selected innovations and forecast stock prices with the best fitted asymmetric.

2. LITERATURE REVIEW

Modelling of stock price volatility has been known to be carried out with the famous GARCH models, out of which several variants of the model emanated to deal with different aspects of stock price volatility. In the modelling of volatility of stock prices, one of the foremost arguments put forward is whether the data possess symmetric or asymmetric effects (Gallo & Pacini, 1998). This consideration extensively affects the choice of models to be used in modelling the particular stock price data.

Numerous techniques have been used in the analysis of the volatility of stock prices, such as: Nugroho *et al.*, (2023) studied the exponential transformation to the exogenous variables in the GARCH-X(1,1) model. The model assumes that the returns error is normally and Student-t distributed. Empirical analysis was carried out based on stock price index data FTSE100 and SP500 daily period from January 2000 to December 2021. The Akaike Information Criterion (AIC) value indicates that the proposed model outperforms the basic GARCH-X(1,1) model, where the best fit model is given by the Student-t distributed model. Emenyonu *et al.*, (2023) this study estimated both the symmetric and asymmetric volatility models. The ARMA-GARCH, ARMA-EGARCH models were employed with the error distributions such as normal distribution, student t-distribution and skewed student-t distribution. The ARMA (2,1)-EGARCH (1,1) with student t-distribution was seen to be the most appropriate model. Chaovanapoonphol *et al.*, (2023) this study proposed an alternative model to analyze the major factors in terms of internal and external factors, which are expected to affect price volatility simultaneously, by applying multiple exogenous Bayesian GARCH-X models. The empirical results of the comparison between the multiple exogenous Bayesian GARCH-X model and the standard Bayesian GARCH-X model, which estimated the impact of individual exogenous variables separately, showed that the standard error of the first model is the smallest compared to the others, which means the multiple exogenous Bayesian GARCH-X model is more fitted to the data than the others.

Moreover, Vaz *et al.*, (2017) this paper empirically showed that (E)GARCH volatility forecasts may be improved by inserting an appropriate exogenous variable in the volatility equation. Several realized measures were tested as regressors and the robust to microstructure effects and/or jumps realized range-based measures provided the best results. Komara, (2020) analyzed relationship between the Prague Stock Exchange Price Index of the Czech Republic (PX Index) and interest rate using ordinary as well as asymmetric GARCH methodology with explanatory variable. Ariyo-Raheem *et al.*, (2023) this study fits an appropriate ARCH/GARCH family model to daily stock returns volatility of each of the selected five most traded assets of the oil and gas marketing companies on the Nigerian stock exchange (NSE), using daily closing prices from January 1, 2005, to December 31, 2020. Using appropriate model selection criteria EGARCH (1,1) with GEDs was found to be the best-fitted models based on the Akaike Information Criterion (AIC). Nguyen & Chaiechi, (2021) explored the effects of natural disasters on the stock market return and volatility in Hong Kong, over various event windows from 1 day to 2 months post disaster. An ARMAX-EGARCHX model, which is an augmentation of a standard ARMA process with an EGARCH process and an intervention variable X, was used for empirical analysis. The results revealed that natural disasters have adverse effects on the Hong Kong Stock Market return and volatility with increasing magnitude.

Similarly, Östermark, (2001) compared a newly developed genetic algorithm (GA) to an extended MEGARCHX algorithm for modelling heteroscedastic vector processes. The competing algorithms are applied to studying the impact of the Japanese stock prices on the Finnish derivatives market. However, the GA provides a powerful supplement to traditional econometric techniques. Zolfaghari & Gholami, (2021) applied a hybrid model which combines adaptive wavelet transform (AWT), long short-term memory (LSTM) and ARIMAX-GARCH family models to predict stock index and combines AWT, LSTM and heterogeneous autoregressive model of realized volatility (HAR-RV) model to predict stock volatility for two major indexes in the U.S. stock market including the Dow Jones Industrial Average (DJIA) and Nasdaq Composite (IXIC). The results indicate an overall improvement in forecasting of stock index using the AWT-LSTM-ARMAX-FIEGARCH model with student's t distribution as compared to the benchmark models. D. B. Nugroho, (2019) in this study, a new class of GARCH models was proposed by applying Yeo-Johnson transformation to the return series. The proposed model was estimated by employing adaptive random walk Metropolis (ARWM) method in Markov chain Monte Carlo (MCMC) scheme. Empirical results on the GARCH(1,1) models showed that the proposed model outperformed the initial model in fitting ten different international stock indices.

Nguyen & Nguyen, (2019) aimed to measure stock price volatility on Ho Chi Minh stock exchange (HSX) by applying symmetric models (GARCH, GARCH-M) and asymmetry (EGARCH and TGARCH) to measure stock price volatility on HSX. The results showed that GARCH (1,1) and EGARCH (1,1) models are the most suitable models to measure both symmetry and asymmetry volatility level of VN-Index. Jimoh & Benjamin, (2020) examined the nexus between the two key economic and financial variables (exchange rate and stock market price) and the most traded crypto currency (Bitcoin and Etherum) in Nigeria. GARCH(1,1), EGARCH(1,1), and Granger causality technique were used to estimate the reaction of the volatility of exchange rates and stock market prices to volatility in crypto currency prices. Mohammed *et al.*, (2022) studied the volatility of equity returns for two beverages traded on the Nigerian stock exchange. The ARCH effect test demonstrated that the two beverages disprove the claim that there is no ARCH effect. According to the preliminary analysis, both beverages were volatile. CGARCH and EGARCH were chosen as the best volatility models for Guinness Nigeria Plc returns and Nigeria Breweries returns, respectively, Fateye *et al.*, (2022) examined the volatility of the daily market price of listed property stocks on the Johannesburg Stock Exchange (JSE) using GARCH(1,1).

In addition, Marobhe, (2020) Modeled both symmetrical and asymmetrical GARCH models; The findings showed that all three models were significant to forecast stock returns volatility at DSE. P-GARCH (1,1) was found to be more accurate in predicting stock returns based on both the RMSE and TIC. Yeasin *et al.*, (2020) introduce and implement an improved GARCH-X model which can account for the effects of influencing factors (X) both into the mean and variance equation simultaneously of the standard GARCH model. Pole & Cavusoglu, (2021) investigated the effect of macroeconomic factors on stock return in the Nigerian stock market using Autoregressive Distributed Lag (ARDL) method of analysis. Findings revealed that money supply and aggregate industrial production positively and significantly affect stock return ($\beta = 0.466098$, $P < 0.05$; $\beta = 0.213141$, $P < 0.05$) while exchange and inflation rates negatively affect stock return in the Nigerian stock exchange market ($\beta = -0.009285$, $P < 0.05$; $\beta = -0.028260$, $P < 0.05$) respectively. The study concludes that macroeconomic factors significantly affect stock return in the Nigerian stock market at short run and long run.

3. RESEARCH METHODOLOGY

The study employed a secondary data covering daily price from 30th January, 2012 to 3rd October, 2024 sourced from www.investing.com. The returns were computed using:

$$y_t = \log p_t - \log p_{t-1}$$

Financial time series offer a more comprehensive analytical approach to financial asset analysis, including stocks. The models used in the analysis are discussed below:

3.1 The GARCH Model

An extension of the ARCH model is the generalized ARCH (GARCH) model. While the ARCH model is straightforward, it frequently needs a large number of parameters to accurately capture the volatility process of an asset return (Ariyo Raheem *et al.*, 2023). The generalized ARCH (GARCH) model is formulated as follows:

Given equation $r_t = \mu_t + a_t$, as the mean, then, the generalized ARCH (GARCH) volatility model is given as;

$$\sigma_t^2 = \propto_0 + \sum_{i=1}^{m} \propto_\mathrm{i} \sigma_{t-1}^2 + \sum_{j=1}^{m} \beta_\mathrm{j} \sigma_{t-j}^2 \tag{1}$$

Where;

a_t, $= \sigma_t\ \varepsilon_t$, and $\{\varepsilon_t\}$ is a sequence of independent and identically distributed (iid) random variables with mean 0 and variance 1.

$$\propto_0 > 0, \propto_1 \geq 1, \beta_j \geq 0 \text{ and } \sum_{i=1}^{\max(m,s)} (\propto_i + \beta_i) < 1.$$

As before, ε_5 is often assumed to follow a standard normal or standardized Student-*t* distribution or generalized error distribution. The $\propto_i$ and $\propto_j$ are referred to as ARCH and GARCH parameters, respectively. The GARCH (1,1) model is given as;

$$\sigma_t^2 = \propto_0 + \propto_1 \sigma_{t-1}^2 + \beta_1\ \sigma^2_{\mathrm{t}-1} \tag{2}$$

Subject to the following conditions:

$$0 \leq \propto_1, \beta_\mathrm{j} \leq 1, (\propto_1 + \beta_1)<1$$

3.2. The Exponential GARCH Model

The exponential ARCH (EGARCH) model was proposed by Nelson, (1991) to overcome some weaknesses of the GARCH model in handling leverage/asymmetric effect associated with financial time series in the volatility spillovers between the New York, Tokyo and London stock markets (Östermark & Höglund, 1997). In particular, unlike the standard GARCH model, the EGARCH model can capture size effects as well as sign effects of stocks. The variance equation of the EGARCH model is given as;

$$\ln(\sigma_t^2) = \propto_0 + \sum_{i=1}^{m} \left\{ \propto_i \left| \frac{a_{t-1}^2}{\sigma_{t-1}} \right| + \gamma_i \left(\frac{a_{t-1}^2}{\sigma_{t-1}} \right) \right\} + \sum_{j=1}^{s} \beta_j \ln)(\sigma_{t-j}^2) \tag{3}$$

$a_{t-i} > 0$ and $a_{t-i} < 0$ implies good news and bad news and their total effects are $(1+\gamma_i)[a_{t-i}]$ and $(1-\gamma_i)[a_{t-i}]$ respectively. When, $\gamma_i < 0$, the expectation is that bad news would have higher impact on volatility. The EGARCH model achieves covariance stationarity when $\Sigma^{s}_{j=1}\ \beta_j < 1$. Based on sthis representation, some properties of the EGARCH model can be obtained in a similar manner as those of the GARCH model.

3.3 Error Distributions

The probability distribution of stock returns often exhibits fatter tails than the standard normal distribution. The existence of heavy-tailedness is probably due to a volatility clustering in stock markets. In addition, another source for heavy-tailedness seems to be the sudden changes in stock returns. An excess kurtosis also might be originated from fat-tailedness. Mostly, in practice, the returns are typically negatively skewed (Nugroho *et al.*, 2023). In order to capture this phenomenon, the normal distribution, student-*t*, and skewed student-*t* distributions are also considered in this analysis.

In the case of *t*-distributions, the volatility models considered are estimated to maximize the likelihood function of a Student's t distribution and skewed Student's t distribution respectively as:

$$L_{(std)}(\theta_t) = -\frac{1}{2}\ln\left(\frac{\pi(d)\Gamma\left(\frac{d}{2^2}\right)}{r\left[\left(d+\frac{1}{2}\right)\right]^2}\right) - \frac{1}{2}\ln S_t^2 - \frac{d+1}{2}\left(1+\frac{(r_t - X_{t,\theta})^2}{S_t^2(d-2)}\right) \tag{7}$$

For the GARCH models characterized by GED, student-t, and skewed student-t distributions which help to capture additional skewness and kurtosis in the returns, which are not adequately captured by Normal error distribution. The GED is estimated by maximizing the likelihood function below:

$$L(\theta_t) = -\frac{1}{2}\sum_{t-1}^{T}\left(\ln 2\pi + \ln\sigma_t^2 + \frac{a_t^2}{\sigma_t^2}\right) \tag{8}$$

In the case of the skewed student’s *t*-innovation, the volatility models is to maximize the likelihood function of the skewed student’s *t* distribution. The probability density function of the skewed student's distribution is

$$f(x) = \begin{cases} \frac{bc}{w}\left(1+\frac{1}{v}\left(\frac{b(x-\varepsilon)}{w}\right)^2\right)^{-\frac{v+1}{2}}, \text{if } x \geq \varepsilon \\ \frac{bc}{w}\left(1+\frac{1}{v}\left(\frac{b(x-\varepsilon)}{w}\right)^2\right)^{-\frac{v+1}{2}}, \text{if } x < \varepsilon \end{cases} \tag{10}$$

The log of the likelihood function is given as follow: For $x \geq 0$:

$$\log L(x;v,\lambda) = \log\left(\frac{2}{v+1} t\left(\frac{x}{v}\right) T\left(\lambda \cdot \frac{x}{v+1}, v+1\right)\right)$$

$$= \log\left(\frac{2}{v+1}\right) + \log\left(t\left(\frac{x}{v}\right)\right) + \log\left(T\left(\lambda \cdot \frac{x}{v+1}, v+1\right)\right) \tag{11}$$

Log-likelihood for $x < 0$:

$$\log L(x;v,\lambda) = \log\left(\frac{2}{v+1} \cdot t\left(\frac{-x}{v}\right)\left[T-1\left(\lambda \cdot \frac{x}{v+1}, v+1\right)\right]\right)$$

$$= \log\left(\frac{2}{v+1}\right) + \log\left(t\left(\frac{-x}{v}\right)\right) + \log\left[T-1\left(\lambda \cdot \frac{x}{v+1}, v+1\right)\right] \tag{12}$$

3.4 Source of Data

The study employed a secondary data covering daily price from 30th January 2012 to 3rd October 2024 sourced from www. Investi.com. The returns were computed using the equation below.

$$r_t = \log p_t - \log p_{t-1} \tag{13}$$

3.5 Model Selection

Model selection was done using information criteria, and the model with the least information criteria value across the error distributions is adjudged the best fitted. If the number of parameters in the model is denoted as p, then the AIC is defined by:

$$AIC\ (p) = -2\ln(Ml\ ) + 2p \tag{14}$$

Where:

Ml is the maximum likelihood estimate.

The Bayesian information criteria (BIC)given by

$$BIC\ (p) = -2\ln(Ml\ ) + p\ \ln\ (N) \tag{15}$$

4. RESULTS AND DISCUSSION

The section presents the result and discussion of our findings. The results are presented in tables and figures below

Table 1: Descriptive Statistics of the Daily Return Series of Nigeria Stock Prices

N	*Mean*	*Max*	*Min*	*Std. Dev*	*Skewness*	*Kurtosis*	*Jarque-Bera*	*Prob.*
3137	0.0001	0.0406	0.0271	0.0044	0.5150	10.701	7891.81	0.0000

Table 1 showed a descriptive statistic of the returns for Nigeria stock price for the period 2012 to 2024. The daily return series of Nigerian stock prices, based on 3,137 observations, showed a slightly positive average return of 0.000188, with a maximum return of 0.040650 and a minimum of –0.027163, indicating some volatility. The standard deviation is low at 0.004409, suggesting a stable market environment. The skewness of 0.515040 reflects a moderate positive skew, while the high kurtosis of 10.70170 indicates a distribution with a sharp peak and heavy tails, pointing to the potential for extreme returns. Furthermore, the Jarque-Bera statistic of 7891.813, with a p-value of 0.000000, confirmed a significant deviation from normality, highlighting the complexities in analyzing this data.

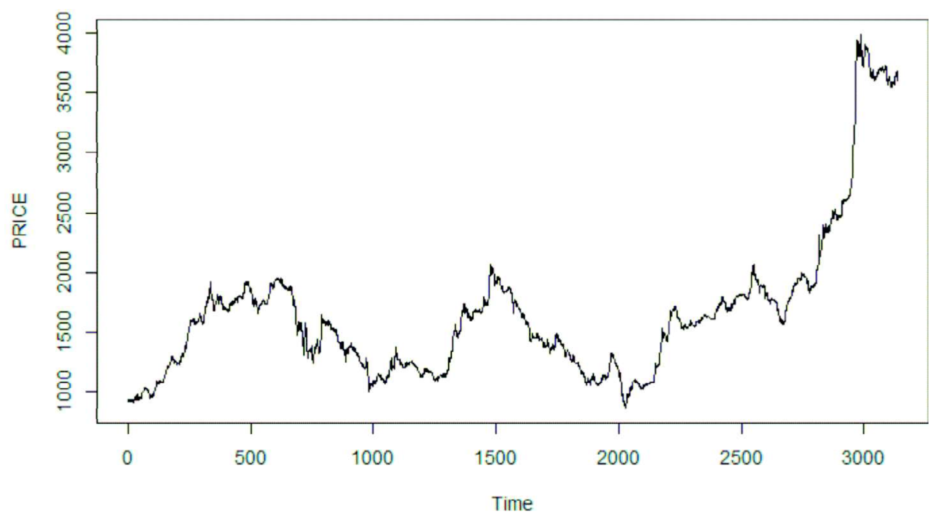

Figure 1: Time Plot of the Nigeria Daily Stock Prices 2012-2024

Figure 1 is a time plot of daily Nigerian stock price from 30th January, 2012 to 3rd October, 2024. The plot shows an upward trend with some slight seasonal variation.

Table 2: Unit Root Test for the Returns on Daily Stock Price

	Test Statistic	*P-Value*	*Level of Significance*
Augmented Dickey Fuller Test	–12.766	0.01	
Phillips-Perron Unit Root Test	–41.229	0.01	0.05

Table 2 presented the results of unit root tests conducted on the return of daily stock prices using the Augmented Dickey-Fuller (ADF) test and the Phillips-Perron (PP) test. The ADF test statistic is -12.766, with a *p*-value of 0.01, indicating strong evidence against the null hypothesis of a unit root, suggesting that the daily stock price returns are stationary. Similarly, the Phillips-Perron test shows a test statistic of –41.229 and a *p*-value of 0.01, further confirming the absence of a unit root. Together, these results imply that the daily stock price returns do not exhibit trends or seasonality over time, making them suitable for various financial modeling and forecasting techniques that assume stationarity.

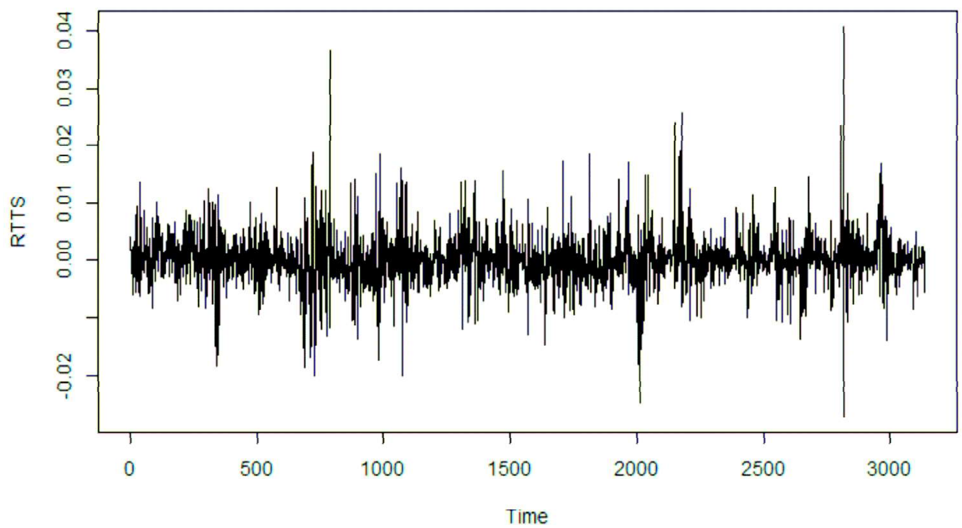

Figure 2: Time Plot of the Return Series for Nigeria Daily Stock Prices

Figure 2 showed the plot of the return series for Nigeria stock price. The plot shows some up and down movement which as an indication of volatility clustering. This is suggesting the presence of ARCH on the residual of the time series. The table showed an ARCH LM test to further

Table 3: ARCH LM-Test: Null Hypothesis: No ARCH Effects

	ARCH LM-Test
Chi-squared	370.93
Df	12
p-value	2.2*e*-16

The ARCH LM test in table 3 assesses whether there are ARCH effects in the time series. With a Chi-squared value of 370.93, 12 degrees of freedom, and a very small *p*-value (2.2*e*-16), the null hypothesis of no ARCH effects is strongly rejected. This indicates that the variance of the error terms is not constant over time, suggesting the presence of ARCH effects in the data. Consequently, models like ARCH or GARCH may be more suitable to capture the time-varying volatility in the series.

Table 4: Estimate of Parameter for EGARCH (1,1) with Student' T Innovation

Parameters	*Estimate*	*Std. Error*	*t value*	*Pr(>|t|)*
μ	0.000033	0.000024	1.38730	0.16535
ω	–0.814765	0.190838	–4.26941	0.00002
α_1	–0.004105	0.018472	–0.22222	0.82414
β_1	0.925503	0.017365	53.29678	0.00000
γ_1	0.461342	0.050839	9.07463	0.00000
Shape	3.331676	0.232944	14.30249	0.00000
		Robust Standard Errors		
μ	0.000033	0.000016	2.02961	0.042396
2	–0.814765	0.294026	–2.77107	0.005587
α_1	–0.004105	0.020984	–0.19562	0.844906
β_1	0.925503	0.026843	34.47828	0.000000
γ_1	0.461342	0.068780	6.70752	0.000000
shape	3.331676	0.259565	12.83561	0.000000

The table presents estimates for various parameters of a statistical model, along with their standard errors, *t*-values, and *p*-values. Notably, parameters such as omega, beta 1, gamma 1, and shape show strong statistical significance, indicated by *p*-values of 0.00002, 0.00000, 0.00000, and 0.00000, respectively. In contrast, *mu* ($p = 0.16535$) and alpha 1 ($p = 0.82414$) are not statistically significant, suggesting that these parameters do not have a meaningful impact on the model.

The robust standard errors also confirm the significance of omega, beta 1, gamma 1, and shape, with adjusted *p*-values further reinforcing their importance. Specifically, beta 1 and gamma 1 indicate strong influences on the model, likely reflecting volatility dynamics, while the shape parameter suggests important distribution characteristics. Overall, the findings highlight key elements of the model that warrant further attention, particularly in understanding volatility and its implications in the context of the data being analyzed.

Table 5: EGARCH (1,1) with Normal Innovation

Parameters	*Estimate*	*Std. Error*	*t value*	*Pr(>|t|)*
μ	0.000142	0.000064	2.2408	0.025037
ω	–1.335895	0.187928	–7.1085	0.000000
α_1	0.034184	0.013054	2.6186	0.008830
β_1	0.876655	0.017015	51.5221	0.000000
γ_1	0.392217	0.028554	13.7359	0.000000
		Robust Standard Errors		
μ	0.000142	0.000090	1.5740	0.115484
ω	–1.335895	0.368500	–3.6252	0.000289
α_1	0.034184	0.025031	1.3657	0.172045
β_1	0.876655	0.033614	26.0804	0.000000
γ_1	0.392217	0.051277	7.6489	0.000000

The EGARCH (1,1) model in the table estimates various parameters to explain volatility dynamics in a time series. The mean (μ) is small but statistically significant at 0.025 with the standard errors, indicating a slight positive return, though its significance becomes questionable when using robust standard errors (p-value of 0.115). The constant term (ω) is highly significant with both standard and robust errors ($p < 0.001$), showing that the base volatility level is an important factor. The ARCH effect (α_1), which captures the impact of past volatility shocks, is significant with regular standard errors ($p = 0.0088$) but loses significance with robust standard errors ($p = 0.172$), suggesting potential volatility clustering.

The GARCH effect (β_1), representing the persistence of volatility, is highly significant ($p = 0.000$) with both standard and robust errors, implying that past volatility plays a strong role in determining future volatility. The leverage effect (γ_1), which accounts for the asymmetric impact of negative shocks on volatility, is also highly significant with both sets of errors, confirming that negative news increases volatility more than positive news. Overall, the model suggests strong volatility persistence and asymmetry, although the influence of past shocks and the mean return may be less robust when adjusting for heteroskedasticity.

Table 6: EGARCH (1, 1) with Skewed Student' *t* Innovation

Parameter	*Estimate*	*Std. Error*	*t value*	*Pr(>|t|)*
μ	0.000071	0.000048	1.45546	0.145542
ω	–0.801447	0.191643	–4.18199	0.000029
α_1	–0.006889	0.018674	–0.36893	0.712177
β_1	0.926758	0.017441	53.13725	0.000000
γ_1	0.456476	0.051211	8.91371	0.000000
Skew	1.021246	0.022293	45.81011	0.000000
Shape	3.341936	0.234530	14.24952	0.000000
		Robust Standard Errors		
μ	0.000071	0.000053	1.32157	0.186313
ω	-0.801447	0.296696	-2.70124	0.006908
α_1	-0.006889	0.021960	-0.31372	0.753730
β_1	0.926758	0.027082	34.22033	0.000000
γ_1	0.456476	0.070190	6.50339	0.000000
Skew	1.021246	0.021549	47.39276	0.000000
Shape	3.341936	0.260527	12.82760	0.000000

The EGARCH (1,1) model with skewed Student's t-distribution innovations reveals several important aspects of volatility dynamics. The mean return (μ) is not statistically significant, indicating that the average return over the period is close to zero. The constant term (ω) is highly significant, suggesting that there is a substantial baseline level of volatility. The ARCH effect (α_1) is not significant, implying that past shocks do not have a strong short-term influence on current volatility. On the other hand, the GARCH effect (β_1) is highly significant, showing that volatility is persistent over time, with past volatility having a strong impact on future volatility levels.

The leverage effect (γ_1) is also significant, indicating that negative shocks increase volatility more than positive ones. Additionally, the significant skewness and shape parameters suggest that the residuals are asymmetric and have fat tails, meaning the model accounts for non-normalities like large jumps or outliers. Even with robust standard errors, the key findings regarding volatility persistence, leverage, and the skewed, fat-tailed nature of the distribution remain robust, making this model well-suited for financial time series analysis.

Table 7: Goodness of Fit Test

Models	*AIC*	*BIC*	*SIC*	*Hannan-Quinn*	*Log-Likelihood*	*Half-life([1]*
EGARCH (1,1) with Student'	–8.445	–8.4335	–8.445	–8.4409	13252.02	8.9532
T Innovation EGARCH (1,1) with Normal Innovation	–8.2909	–8.2813	–8.2909	–8.2874	13009.27	5.2654
EGARCH (1,1) with Skewed Student's *t* Innovation	–8.445	–8.4311	–8.4446	–8.4398	13252.37	9.1128

Table 7 compared the three EGARCH(1,1) models using different innovations: Student's *t*-distribution, normal innovation, and skewed Student's *t*-distribution. The skewed Student's *t*-distribution model showed the best fit with the highest log-likelihood (13252.37), the lowest AIC (–8.4446), BIC (–8.4311), and SIC (–8.4446) values, and the longest half-life (9.1128). The Student's *t*-distribution model closely follows, with a log-likelihood of 13252.02, AIC of –8.445, BIC of –8.4335, SIC of –8.445, and a half-life of 8.9532.

The normal innovation model performs the worst, with a lower log-likelihood (13009.27) and higher AIC (–8.2909), BIC (–8.2813), SIC (–8.2909), and a shorter half-life (5.2654). This indicates that the *t*-distribution models, particularly the skewed Student's t model, better capture the data's volatility dynamics and offer more persistent volatility shocks. Thus, the EGARCH (1,1) with skewed Student's *t* innovation was used to forecast the returns for the stock price for 10 days ahead as shown in figure 3.

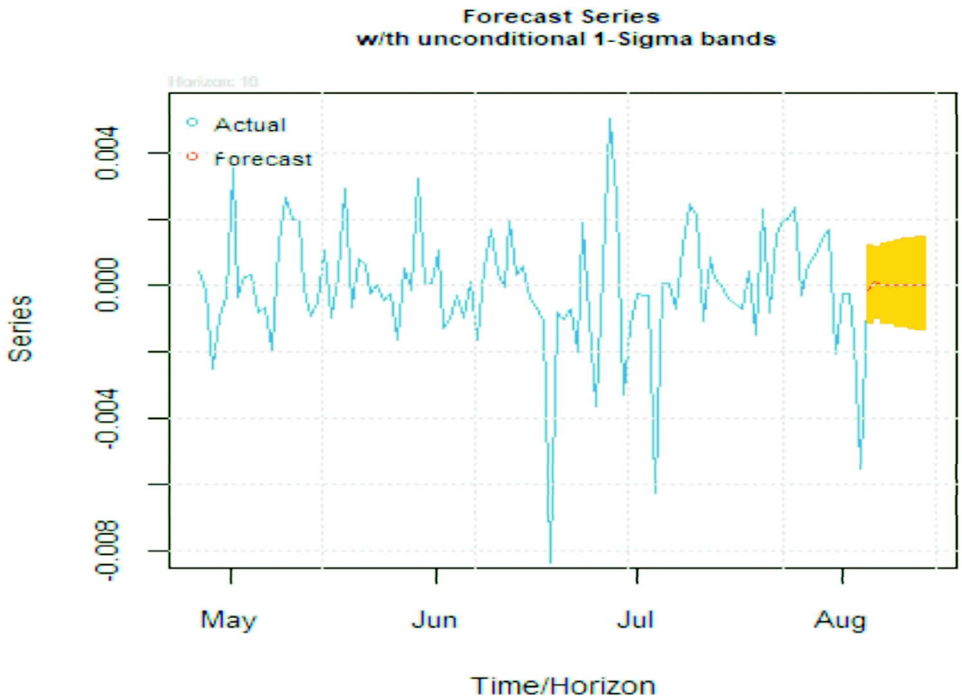

Figure 3: Forecast Plot for 10 Days Period of the Series

Figure 3 is a forecast plot of the Nigerian stock prices using the selected model. The forecast lies within 95% CL.

Table 8: Forecast Horizon

	Forecast
RMSE	0.004413
MAE	0.002907
Theil Inequality Coefficient	0.321566
Bias proportion	0.002922
Variance Proportion	0.0001810

The findings provide a comprehensive analysis of the volatility dynamics and stationarity properties of the time series data, particularly for daily stock returns. The ARCH LM test strongly rejects the null hypothesis of no ARCH effects, with a Chi-squared value of 370.93 and an extremely low *p*-value (2.2e-16), indicating that the variance of the residuals fluctuates over time. This result highlights the presence of autoregressive conditional Heteroscedasticity (ARCH) effects, which is typical in financial time series. The implication is that models like ARCH or GARCH are needed to appropriately model time-varying volatility. The EGARCH (1,1) models highlight the differences in how different innovations (normal, Student's *t*, and skewed Student's *t*-distributions) capture volatility dynamics. The skewed Student's t-distribution model provides the best overall fit, with the lowest AIC (–8.4446) and the highest log-likelihood (13252.37). This model captures the asymmetric nature of volatility, showing that negative shocks increase volatility more than positive ones (leverage effect) which is agreement with Mohammed *et al.*, (2022). Both the ARCH and GARCH components reflect volatility clustering and persistence, with significant coefficients for β_1 and γ_1 across the models. Moreover, the unit root tests (ADF and PP) confirm that the return series is stationary, making the data suitable for financial forecasting techniques that rely on this property. The findings indicate that volatility is persistent, with fat tails and skewness in the distribution, which makes models like EGARCH (1,1) with skewed Student's *t* innovations well-suited for explaining the behavior of stock returns.

5. CONCLUSION

The analysis reveals important characteristics of daily stock return data, emphasizing the presence of time-varying volatility through significant ARCH effects. The ARCH LM test confirms the need for advanced volatility models like EGARCH to capture the non-constant variance over time. Among the models considered, the EGARCH (1,1) model with skewed Student's *t*-distribution emerges as the most robust, effectively capturing both volatility persistence and the asymmetric response to market shocks, with negative news leading to higher volatility. This is in line with Emenyonu *et al*., (2023). Additionally, the unit root tests demonstrate that the stock return series is stationary, supporting the use of time series models for financial forecasting. Overall, these findings highlight the importance of accounting for volatility dynamics, fat tails, and skewness in modeling stock returns, making the EGARCH (1,1) model with skewed Student's *t*-distribution an optimal choice for analyzing financial time series data.

ACKNOWLEDGMENTS

The authors wish to express their gratitude to the Management of National Mathematical Centre (NMC), Abuja, Nigeria for the opportunity given to the authors to participate in the Research Oriented Course (ROC) in Time Series Analysis & Applications.

REFERENCES

Ariyo Raheem, M., Domingo Mbeke, R., & John Inyang, E. (2023). Volatility Modelling of Stock Returns of Selected Nigerian Oil and Gas Companies. *Science Journal of Applied Mathematics and Statistics*, 11(2), 26-36. https://doi.org/10.11648/j.sjams.20231102.11

Chaovanapoonphol, Y., Singvejsakul, J., & Wiboonpongse, A. (2023). Analysis of Exogenous Factors to Thailand Coffee Price Volatility: Using Multiple Exogenous Bayesian GARCH-X Model. *Agriculture (Switzerland)*, 13(10). https://doi.org/10.3390/agriculture13101973

Emenyonu, S. C., Osu, B. O., & Olunkwa, C. (2023). Estimating Volatility of Daily Price Returns of Nigerian Stock Market. *Journal of Mathematical Techniques and Computational Mathematics*, 2(4), 163-169. https://doi.org/10.33140/jmtcm.02.04.01

Fateye, O. B., Ajayi, O., & Ajayi, C. A. (2022). Modelling of Daily Price Volatility of South Africa Property Stock Market Using GARCH Analysis. *Journal of African Real Estate Research*, 7(2), 24-42. https://doi.org/10.15641/jarer.v7i2.1144

Gallo, G. M., & Pacini, B. (1998). Early News is Good News: The Effects of Market Opening on Market Volatility. *Studies in Nonlinear Dynamics & Econometrics*, 2(4). https://doi.org/10.2202/1558-3708.1034

Jimoh, S. O., & Benjamin, O. O. (2020). The Effect of Cryptocurrency Returns Volatility on Stock Prices and Exchange Rate Returns Volatility in Nigeria. *Acta Universitatis Danubius*, 16(3), 200-213. Retrieved from https://doaj.org/article/3afc1a895317415ab0feabb6d64ae2dc

Khan, K., & Abdullah, U. M. (2019). *Modelling Volatility in stock prices using ARCH / GARCH Technique" Modeling Volatility in Stock Prices using ARCH / GARCH.* (June 2018).

Komara, S. (2020). *GARCH Type Models to Find Linkages between Stock Returns and Interest Rate in The Czech Republic 1 Introduction 2 Literature Review.* 49, 270-281.

Korn, R., & Erlwein-sayer, C. (2013). *GARCH-Extended Models?: Theoretical Properties and Applications Introduction / Motivation.*

Koutmos, G., & Booth, G. G. (1995). Asymmetric Volatility Transmission in International Stock Markets. *Journal of International Money and Finance*, 14(6), 747-762. https://doi.org/10.1016/0261-5606(95)00031-3

Marobhe, M. (2020). *Modeling Stock Market Volatility Using GARCH Models Case Study of Dar es Salaam Stock Exchange (DSE).* 9(2), 138-150.

Mohammed, T., Umar, Y. H., & Adams, S. O. (2022). Modeling the Volatility for Some Selected Beverages Stock Returns in Nigeria (2012-2021): a Garch Model Approach. *Matrix Science Mathematic*, 6(2), 41-51. https://doi.org/10.26480/msmk.02.2022.41.51

Nelson, D. B. (1991). Conditional Heteroskedasticity in Asset Returns: A New Approach. *Econometrica*, 59(2), 347. https://doi.org/10.2307/2938260

Nguyen, C. T., & Nguyen, M. H. (2019). Modeling stock price volatility: Empirical evidence from the Ho Chi Minh City stock exchange in Vietnam. *Journal of Asian Finance, Economics and Business*, 6(3), 19-26. https://doi.org/10.13106/jafeb.2019.vol6.no3.19

Nguyen, T., & Chaiechi, T. (2021). The Effects of Natural Disasters on Stock Market Return and Volatility in Hong Kong. *Economic Effects of Natural Disasters: Theoretical Foundations, Methods, and Tools*, 11-20. https://doi.org/10.1016/B978-0-12-817465-4.00002-9

Nugroho, D. B. (2019). GARCH(1,1) model with the Yeo-Johnson transformed returns. *Journal of Physics: Conference Series*, 1320(1). https://doi.org/10.1088/1742-6596/1320/1/012013

Nugroho, D., Dimitrio, O., & Tita, F. (2023). The GARCH-X(1,1) Model with Exponentially Transformed Exogenous Variables. *JST (Jurnal Sains Dan Teknologi)*, 12(1), 65-72. https://doi.org/10.23887/jstundiksha.v12i1.50714

Östermark, R. (2001). Genetic modelling of multivariate EGARCHX-processes: evidence on the international asset return signal response mechanism. *Computational Statistics & Data Analysis*, 38(1), 71-93. https://doi.org/10.1016/S0167-9473(01)00028-7

Östermark, R., & Höglund, R. (1997). Multivariate EGARCHX-modelling of the international asset return signal response mechanism. *International Journal of Finance and Economics*, 2(3), 249-262. https://doi.org/10.1002/(SICI)1099-1158(199707) 2:3<249::AID-IJFE47>3.0.CO;2-4

Pole, H., & Cavusoglu, B. (2021). The Effect of Macroeconomic Variables on Stock Return Volatility in the Nigerian Stock Exchange Market. *Asian Journal of Economics, Finance and Management*, 3(3), 32-43.

Vaz, B., Mendes, D. M., & Bello, V. (2017). Improving (E) GARCH forecasts with robust realized range measures?: Evidence from international markets. *Journal of Economics and Finance.* https://doi.org/10.1007/s12197-017-9386-x

Yeasin, M., Narain Singh, K., Lama, A., & Kumar Paul, R. (2020). Modelling Volatility Influenced by Exogenous Factors using an Improved GARCH-X Model. *Journal of the Indian Society of Agricultural Statistics*, 74(3), 209-216. Retrieved from https://www.researchgate.net/publication/348163301

Zakaria, S., Abdalla, S., & Winker, P. (2017). Modelling Stock Market Volatility Using Univariate GARCH Models?: Evidence Modelling Stock Market Volatility Using Univariate GARCH Models?: Evidence from Sudan and Egypt. (July 2012). https://doi.org/10.5539/ijef.v4n8p161

Zolfaghari, M., & Gholami, S. (2021). A Hybrid Approach of Adaptive Wavelet Transform, Long Short-Term Memory and ARIMA-GARCH Family Models for the Stock Index Prediction. *Expert Systems With Applications*, 182, 115149. https://doi.org/10.1016/J.ESWA.2021.115149

for citation :

Adenomo M.O., Awogbemi C.A., & et al. (2024). Asymmetry Volatility Modeling of Nigerian Stock Prices with Some Selected Distribution of Innovations. *Journal of Applied Econometrics and Statistics,* Vol. 3, No. 1-2, pp. 79-95.